APPLICATION OF DERIVATIVE:

(1) A man, 2 m tall, walks at the rate of $1\frac{2}{3}\ m/s$ towards a street light which is $5\frac{1}{3}$ m above the ground. At what rate is the tip of his shadow moving? At what rate is the length of shadow changing when he is $3\frac{1}{3}\ m$ from the base of the light?

(2) A ladder 0.05 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 20 cm/min. How fast is its height on the wall decreasing when the foot of the ladder is 0.04 m away from the wall?

(3) Show that $y = \log(1 + x) - \frac{2x}{2+x}$, $x > -1$, is an increasing of x throughout its domain.

(4) Find the intervals in which the function f given by $f(x) = \sin x - \cos x$, $0 \le x \le 2\pi$ is strictly increasing or strictly decreasing. Hence also find maximum (local maiximum) and minimum (local minimum) value and find difference greatest (absolute maxima) and least value (absolute minima) on interval $[0, 2\pi]$.

(5) Find the value of x for which $y = [x(x-2)]^2$ *is an increasing function*. Hence also find points parallel to x – axis.

(6) Prove that $y = \frac{4\sin\theta - 2\theta - \theta\cos\theta}{2+\cos\theta}$ is an increasing function of θ in $\left[0, \frac{\pi}{2}\right]$. Hence also find sub interval on $[0, 2\pi]$.

(7) Find the condition that the curves $2x = y^2$ and $2xy = k$ intersect orthogonally. $k^2 = 8$

(8) (i) Show that the line $\frac{x}{a} + \frac{y}{b} = 1$ touches the curve $y = b\,e^{-x/a}$ at the point where crosses the y – axis

(ii) If $y(x-4)(x-5) = (x-7)$ cuts the x – axis find equation of tangent.

(9) Find equation of normal to the curve $x^2 = 4y$ which passes through the point (1, 2). Also find corresponding tangent.

(10) Show that the normal at any point **θ** to the curve $x = a\cos\theta + a\theta\sin\theta$, $y = a\sin\theta - a\cos\theta$ at a constant distance from the origin. Also find $\frac{d^2y}{dx^2}$ $at\ x = \frac{\pi}{4}$

(11) Find the coordinates of the point on the curve $\sqrt{x} + \sqrt{y} = 4$ at which tangent is equally inclined to the axes.

(12) Find the points at which the function *f* given by $f(x) = (x-2)^4 (x+1)^3$ has

(i) local maxima **(ii)** local minima **(iii)** point of inflexion

(13) (a) An Apache helicopter of enemy is flying along the curve given by $y = x^2 + 7$. A soldier, placed at (3, 7), wants to shoot down the helicopter when it is nearest to him. Find the nearest distance (called also shortest distance).

(b) find the shortest distance curve $y = x^2$ from the line $y - x = 1$.

(14) If the sum of length of the hypotenuse and a side of a right angled triangle is given, show that the area of triangle is maximum, when the angle between is $\frac{\pi}{3}$.

(15) (a) Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical angle 30^0 is one-third that of the cone and the greatest volume of cylinder is $4/81\pi h^3$.

(b) Show that the curved surface of a right circular cylinder inscribed in a right circular cone is maximum when the radius of its base is half that of the cone.

(16) (a) Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is 8/27 of the volume of the sphere.

(b) Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $2R/\sqrt{3}$. Also find the maximum volume, if $R = 5\sqrt{3}$.

(17) If lengths of three sides of a trapezium other than base are equal to 10cm, then find the area of the trapezium when it is maximum.

(18) (i) Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ time the radius of the base. Or $h = \sqrt{2}r$ *or* $\cot^{-1}\sqrt{2}$

(ii) Prove that the semi – vertical angle of a right circular cone of maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$ OR $\sin^{-1}\sqrt{\frac{2}{3}}$ *OR* $\cos^{-1}\frac{1}{\sqrt{3}}$

(19) AB is a diameter of a circle and C is any point on the circle. Show that the area of Δ ABC is maximum, when it is isosceles.

INTEGRATION:

(20) Evaluate: $\int e^{\tan^{-1}x}\left(\frac{1+x+x^2}{1+x^2}\right)dx$ **(21)** Evaluate: $\int \frac{x^2}{x^4-x^2-12}dx$

(22) Prove that $\int \frac{5x+3}{\sqrt{x^2+4x+10}}dx$ **(23)** Evaluate: $\int (3-2x)\sqrt{2+x-x^2}\,dx$

(24) Evaluate: $\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}dx$ **(25)** Evaluate $\int \frac{(3\sin x-2)\cos x}{5-\cos^2 x-4\sin x}dx$

(26) Evaluate $\int \frac{1}{\sin x-\sin 2x}dx$ **(27)** Evaluate: $\int \frac{x^4}{(x-1)(x^2+1)}dx$

(28) Evaluate $\int \frac{dx}{\sin^4 x+\cos^4 x}$ OR $\int \frac{x^2+1}{x^4+1}dx$

(29) Prove that $\int_0^1 \log\frac{(1+y)}{1+y^2}dy = \int_0^{\pi/4}\log(1+\tan x)dx = \frac{\pi}{8}\log 2$

(30) Prove that $\int_0^{\pi} \frac{x}{a^2\cos^2 x+b^2\sin^2 x}dx = \frac{\pi^2}{2ab}$ **(31)** Prove that: $\int_0^{\pi/2}\frac{\cos^2 x}{1+3\sin^2 x} = \frac{\pi}{6}$

(32) Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{1+e^x}dx$

(33) Evaluate (a) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx$ (b) $\int_0^{\frac{\pi}{4}}\frac{\sin x+\cos x}{9+16\sin 2x}dx$

(34) Evaluate: $\int_0^{\pi/2}\sin 2x\tan^{-1}(\sin x)\,dx$ **(35)** Evaluate $\int\left[\log(\log x)+\frac{1}{(\log x)^2}\right]dx$

(36) Evaluate: $\int_0^{\pi/2}\frac{x\sin x\cos x}{\cos^4 x+\sin^4 x}dx = \frac{\pi^2}{16}$ **(37)** Evaluate: $\int_{-\pi}^{\pi}\frac{2x(1+\sin x)}{1+\cos^2 x}dx$

(38) Find: $\int_0^2(x^2+e^{2x+1})\,dx$ *as the limit of sum.* **(39)** Evaluate: $\int \frac{x^3}{x^4+3x^2+2}dx$

(40) Evaluate $\int_{-1}^2 f(x)$, *where* $f(x) = |x+1|+|x|+|x-1|$

(41) Evaluate: $\int_0^{\frac{\pi}{2}}\frac{\sin^2 x}{\sin x+\cos x}dx$

APPLICATION OF INTEGRATION:

(42) Find the area of the region parabola $y = \frac{3x^2}{4}$ and the line $3x - 2y + 12 = 0$.

(43) Find the area of the region enclosed by x – axis, line $x = \sqrt{3}y$ and curve $x^2 + y^2 = 4$.

(44) Using integration find the area of region bounded by the triangle whose vertices are (– 1, 0), (1, 3) and (3, 2).
(45) Using integration, find the area bounded by the tangent to the curve $4y = x^2$ at the point (2, 1) and the lines whose equations are $x = 2y$ and $x = 3y - 3$.
(46) Find the area between the curves $|x - 1| \leq y \leq \sqrt{5 - x^2}$.

(47) Find the interior and exterior area bounded by curve $4x^2 + 4y^2 = 9$ and $x^2 = 4y$.
(48) Find the area bounded by the curve $y = \sqrt{x}$ and $x = 2y + 3$ in the first quadrant and the x- axis.
(49) Find the area of the curve $\mathbf{x^2 + y^2 \leq 1 \leq x + \frac{y}{2}}$.
(50) Find the area bounded by the curves $x^2 + y^2 \leq 8x$ and $y^2 \leq 4x$.